Math 165 – Quiz 10C, area between curves – solutions

Problem 1  Find the area of the region in the first quadrant between the x-axis and the curves given by $y = 6x - x^2$ and $y = 12 - 2x$.

Solution  From solving $6x - x^2 = 12 - 2x$

we get $x = 2$ and $x = 6$ as the x-coordinates of points where the two given curves meet. The curve $y = 12 - 2x$ and the x-axis (with equation $y = 0$) intersect at $x = 6$ as well. Interpreting the region as that between the x-axis and the lower of the two curves, we split it into the part $A$ above $[0, 2]$ and the part $B$ above $[2, 6]$.

Region $A$ has area

$$\int_{0}^{2} 12 - 2x \ dx = \left[ 12x - x^2 \right]_{0}^{2} = 20.$$  

Region $B$ has area

$$\int_{2}^{6} 6x - x^2 \ dx = \left[ 3x^2 - \frac{x^3}{3} \right]_{2}^{6} = \frac{80}{3}.$$  

So the total area equals $140/3$. 