Math 165 – Quiz 8C, Local/Global Extrema
– solutions

Problem 1  Consider the function

\[ f(x) = (x^2 + 2x - 7)e^x \]

a) Find all critical points of this function.
b) Determine which of these points is a local minimum, and which is a local maximum.
c) Determine the global (absolute) maximum of this function on the interval \([-2, 2] \).

Solution  a) We first have to find the derivative of \( f(x) \), using Product Rule and Chain Rule.

\[
\begin{align*}
f'(x) & = (2x + 2)e^x + (x^2 + 2x - 7)e^x \\
& = (x^2 + 4x - 5)e^x.
\end{align*}
\]

Then, we solve \( f'(x) = 0 \). This is equivalent to

\[ x^2 + 4x - 5 = 0, \]

which we solve using the quadratic formula or factorization, so \( x = -5 \) and \( x = 1 \) are the critical points of \( f(x) \).
b) The sign pattern of \( f'(x) \) is

\[
\begin{array}{c|c|c|c}
  + & - & + \\
  -5 & 1 & 
\end{array}
\]

which means that \( f(x) \) has a local maximum at \( x = -5 \) and a local minimum at \( x = 1 \).
c) We need to check values of \( f(x) \) at \( x = -2 \) and at \( x = 2 \), which clearly gives that the maximum resides at \( x = 2 \). We can skip checking \( x = 1 \) because the function increases from there, so it can’t give the maximum. On the back is a plot of \( f(x) \) with local extrema – see how hard it is to tell about the local maximum at \( x = -5 \)?
Same function, zooming in on $x = -5$. 