Math 165 – Quiz 7D, Linearization – solutions

Problem 1  For the function

\[ f(x) = \frac{4 - x}{x + 3} \]

we want to approximate \( f(1.996) \).

a) Find the linearization of \( f(x) \) at \( x = a \) where \( a \) is a suitably chosen integer.

b) With your answer from part a), use the linearization to approximate \( f(1.996) \). No credit for exact answers or rounded decimal fractions, use common fractions in your approximation.

Solution  Below, I will give both common fractions and (un-rounded) decimal fractions for reference.

a) First, we differentiate \( f(x) \) using the Quotient Rule:

\[ f'(x) = \frac{-(x + 3) - (4 - x)}{(x + 3)^2} = -\frac{7}{(x + 3)^2}. \]

We choose \( a = 2 \), so \( f'(a) = -7/25 = -0.28 \) and

\[ L(x) = -\frac{7}{25}(x - 2) + \frac{2}{5} = -0.28(x - 2) + 0.4. \]

b) We just need to plug \( x = 1.996 \) into \( L \).

I will show two alternatives of doing the arithmetic - either with common fractions, or with decimal fractions (no rounding, however!).

\[ L(1.996) = -\frac{7}{25} \cdot \frac{4}{1,000} + \frac{2}{5} = \frac{10028}{25,000} = \frac{40112}{100,000} = -0.28 \cdot (-0.004) + 0.4 = 0.40112. \]