Problem 1  Consider the function $y = f(x)$ given by

$$y = x^3 - \tan(x).$$

a) Find the derivative of the inverse of $f(x)$ at the point $\left(\frac{\pi}{4}\right)^3 - 1$. Note that $f(\pi/4) = (\pi/4)^3 - 1$.

b) Find an equation for the tangent line to the graph of the inverse at the point $((\pi/4)^3 - 1, \pi/4)$.

Solution  a) First, use the Product Rule to get

$$f'(x) = 3x^2 - \sec^2(x).$$

Call the inverse function $g$. Then

$$g'(y) = \frac{1}{f'(x)} = \frac{1}{3x^2 - \sec^2(x)}$$

where $x = g(y)$ (we do not have a formula to compute $g(y)$!). But we know that for $x = \pi/4$, $y = (\pi/4)^3 - 1$, also $\sec(\pi/4) = \sqrt{2}$, so

$$g'(((\pi/4)^3 - 1) = \frac{1}{3 \cdot (\pi/4)^2 - 2} = \frac{16}{3\pi^2 - 32}.$$  

b) Use the answer from part a)! We have the slope of the tangent line. Using $x$ as input and $y$ as output (that means switching $x$, $y$ in the work in part a)),

$$y = \frac{16}{3\pi^2 - 32}(x - (\pi/4)^3 - 1)) + \frac{\pi}{4}.$$