Math 151 A,B,P,Q,W: Calculus for Business and Social Sciences  
Final Exam – SAMPLE

INSTRUCTIONS: Show all work in the space provided. Clearly indicate your final answers. Calculators are allowed. However, answers without work will NOT receive full credit. The maximum possible score is 140 points.

Question 1 (5 points). Solve for $x$.

$\left(\frac{1}{2}\right)^{1-x} = 4.$

$\Rightarrow \left(\frac{1}{2}\right)^{1-x} = 4 \Rightarrow \left(\frac{1}{2}\right) = 2 \Rightarrow 2^{x-1} = 2 \Rightarrow x-1 = 2 \Rightarrow x = 3$

Question 2 (5 points). If $2^x = 3$, what does $4^{-x}$ equal?

$4^{-x} = \left(2^2\right)^{-x} = 2^{2(-x)} = 2^{-2x} = 2^{(x)(-2)} = (2^x)^{-2} = (3)^{-2} = \frac{1}{9}$

Question 3 (5 points). Solve for $x$.

$log_2(2x + 1) = 3$

Use inverse of log function to "dig out the $x$":

$log_2(2x+1) = 3 \Rightarrow 2^3 = 2x+1 \Rightarrow 2x = 7 \Rightarrow x = \frac{7}{2}$
Question 4 (5 points). Use properties of logarithms to write the following expression as the sum and/or difference of logarithms. Express powers as factors.

\[ \log_2 \left( \frac{x^3}{x-1} \right), \ x > 3 \]

\[ \log_2 \left( \frac{x^3}{x-1} \right) = \log_2 x^3 - \log_2 (x-1) \quad \text{(Quotient Rule)} \]

\[ = 3 \log_2 x - \log_2 (x-1) \quad \text{(Power Rule)} \]

(Note: \( \log_2 (x-1) \neq \log_2 x - \log_2 1 \))

Question 5 (5 points). Use properties of logarithms to write the following expression as a single logarithm.

\[ 2 \log_a (5x^3) - \frac{1}{2} \log_a (2x + 3) \]

\[ 2 \log_a (5x^3) - \frac{1}{2} \log_a (2x + 3) = \frac{1}{2} \log_a (5x^3) - \frac{1}{2} \log_a (2x + 3) \quad \text{(Power Rule)} \]

\[ = \log_a \left( \frac{5x^3}{2x + 3} \right) \quad \text{Prop of Exponents} \]

\[ = \log_a 25x^6 - \log_a \sqrt{2x + 3} \]

\[ = \log_a \frac{25x^6}{\sqrt{2x + 3}} \quad \text{(Quotient Rule)} \]

Question 6 (5 points). If a bank pays 9% annual interest compounded continuously, how long will it take for an initial deposit to triple in value?

Continuous Compounding: \[ A = Pe^{rt} \]

\[ \text{Final amount} \quad \text{Initial principal} \quad \text{Annual interest rate (as a %)} \quad \text{Period in years} \]

Let initial principal = \( P_0 \)

Solve:

\[ 3P_0 = P_0 e^{0.09t} \]

\[ \Rightarrow \ln e^{0.09t} = \ln 3 \]

\[ \Rightarrow 0.09t = \ln 3 \]

\[ \Rightarrow t = \frac{\ln 3}{0.09} \text{ yrs} \]

\( \approx 12.2 \text{ yrs} \)
**Question 7 (5 points).** Evaluate the following limit or reply "DNE" if the limit does not exist.

\[
\lim_{x \to 2} \frac{3x^2 + 4}{x^2 + x} = \lim_{x \to 2} \frac{3x + 4}{x + 1} = \frac{3 \cdot \lim_{x \to 2} x + \lim_{x \to 2} 4}{\lim_{x \to 2} x + \lim_{x \to 2} x} = \frac{3(2) + 4}{2 + 2} = \frac{10}{4} = \frac{5}{2}
\]

**Note:** This shows all limit props being used.

**Shortcut:** is "plug in \( x = 2 \)" and make sure denominator is not zero!

**Question 8 (5 points).** Evaluate the following limit or reply "DNE" if the limit does not exist.

\[
\lim_{x \to 3} \frac{x^2 + x - 6}{x^2 + 2x - 3} = \lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)(x+1)}
\]

Try factoring:
\[
\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 + 2x - 3} = \lim_{x \to 3} \frac{(x+3)(x-2)}{(x+3)(x-1)}
\]

\[
= \lim_{x \to 3} \frac{x-2}{x-1} = \frac{-5}{4}
\]

**Question 9 (5 points).** Evaluate the following limit or reply "DNE" if the limit does not exist.

\[
\lim_{x \to \infty} \frac{3x^2 - 1}{4x^2}
\]

By "end behavior" of polynomials:

\[
\lim_{x \to \infty} \frac{3x^2}{4x^2} = \frac{3}{4}
\]

For all \( x \neq 0 \)

**Question 10 (5 points).** Evaluate the following limit or reply "DNE" if the limit does not exist.

\[
\lim_{x \to 2} \frac{1-x}{3x-6}
\]

As \( x \) gets closer to 2 with \( x < 2 \):

\[
1 - x\quad \text{gets closer to} \quad 1 - (2) = -1\quad \text{(a fixed negative number)}
\]

and \( \frac{1}{3x - 6} \) is negative and unbounded.

So, \( \frac{1-x}{3x-6} \) is positive and unbounded.

That is,

\[
\lim_{x \to 2} \frac{1-x}{3x-6} = \infty
\]
Question 11. Let \( f(x) = \begin{cases} 
3x + 1 & \text{if } x \leq 0 \\
-x^2 & \text{if } 0 < x \leq 2 \\
\frac{1}{2}x - 5 & \text{if } 2 < x 
\end{cases} \)

Find the following limits or values. Write "DNE" for a limit or value that does not exist.

(a) (1 point) \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3x + 1) = 3(0) + 1 = 1 \)

\( \text{Top branch because } x < 0 \text{ as } x \to 0^- \)

(b) (1 point) \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (-x^2) = -(0)^2 = 0 \)

\( \text{Middle branch because } x > 0 \text{ as } x \to 0^+ \)

(c) (1 point) \( f(0) = 3(0) + 1 = 1 \)

\( \text{Top branch used when } x < 0 \)

(d) (2 points) Is \( f(x) \) continuous at \( x = 0 \)? Explain your answer.

\[ \boxed{\text{No}} \]

Because continuity at \( x = 0 \) requires that \( \lim_{x \to 0} f(x) = f(0) \) and \( \lim_{x \to 0} f(x) \) DNE because 1-sided limits do not match.

(e) (1 point) \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (-x^2) = -(2)^2 = -4 \)

\( \text{Middle branch} \)

(f) (1 point) \( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left( \frac{1}{2}x - 5 \right) = \frac{1}{2}(2) - 5 = 1 - 5 = -4 \)

\( \text{Bottom branch} \)

(g) (1 point) \( f(2) = -(2)^2 = -4 \)

\( \text{Middle branch when } x = 2 \)

(h) (2 points) Is \( f(x) \) continuous at \( x = 2 \)? Explain your answer.

\[ \boxed{\text{Yes}} \]

Because \( \lim_{x \to 2} f(x) = -4 = f(2) \)

Thus, \( \lim_{x \to 2} f(x) = f(2) \) as required.
Question 12 (5 points). Use the definition of the derivative \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) to find the derivative of the following function. (Do not use the “easy” differentiation rules for this problem.)

\[
\begin{align*}
\frac{d}{dx} (3 - x^2) &= \lim_{h \to 0} \frac{3 - (x + h)^2 - (3 - x^2)}{h} \\
&= \lim_{h \to 0} \frac{3 - (x^2 + 2xh + h^2) - 3 + x^2}{h} \\
&= \lim_{h \to 0} \frac{(-2x - h)}{h} \\
&= -2x
\end{align*}
\]

Question 13. For a certain production facility, the cost function is \( C(x) = 6x + 15 \) and the revenue function is \( R(x) = 24x - 3x^2 \) where \( x \) is the number of units produced and sold, and \( R \) and \( C \) are measured in millions of dollars.

(a) (2 points) Find the marginal revenue function.

\[
\text{Marginal revenue} = R'(x) = \frac{d}{dx} [R(x)] = \frac{d}{dx} [24x - 3x^2] = 24 - 6x
\]

(b) (2 points) Find the marginal cost function.

\[
\text{Marginal cost} = C'(x) = \frac{d}{dx} [C(x)] = \frac{d}{dx} [6x + 15] = 6
\]

(c) (3 points) Find the break-even point(s) [the number(s) \( x \) for which \( R(x) = C(x) \)].

\[
\begin{align*}
R(x) &= C(x) \\
24x - 3x^2 &= 6x + 15 \\
3x^2 - 24x + 6x + 15 &= 0 \\
3x^2 - 18x + 15 &= 0 \\
3(x^2 - 6x + 5) &= 0 \\
3(x - 1)(x - 5) &= 0 \\
&\Rightarrow x = 1 \text{ or } x = 5
\end{align*}
\]

(d) (3 points) Find the number \( x \) for which marginal revenue equals the marginal cost.

\[
R'(x) = C'(x) \Rightarrow 24 - 6x = 6 \\
\text{From a) From b)} \\
6x - 24 + 6 &= 0 \\
6x - 18 &= 0 \\
x &= 3
\]
Section 4.1

Question 14 (5 points). Find the value of \( \frac{dy}{dx} \) at the indicated point.

\[ y = 2 - 2x - x^3 \quad \text{at} \quad (2, 6) \]

\[
\frac{dy}{dx} = \frac{d}{dx} \left[ 2 - 2x - x^3 \right] = 0 - 2 - 3x^2 = -2 - 3x^2
\]

When \( x = 2 \),

\[
\left. \frac{dy}{dx} \right|_{x=2} = -2 - 3(2)^2 = -2 - 3(4) = -2 - 12 = -14
\]

Section 4.2

Question 15 (5 points). Find \( f'(x) \) where \( f(x) = (x^6 - 2)(4x^2 + 1) \).

\[
\begin{align*}
\frac{d}{dx} \left[ (x^6 - 2)(4x^2 + 1) \right] &= (x^6 - 2) \cdot \frac{d}{dx} [4x^2 + 1] + (4x^2 + 1) \cdot \frac{d}{dx} [x^6 - 2] \\
&= (x^6 - 2) \cdot (8x) + (4x^2 + 1) \cdot (6x^5) \\
&= 8x^7 - 16x + 24x^7 + 6x^5 \\
&= 32x^7 + 6x^5 - 16x
\end{align*}
\]

(Fully simplified)

Section 4.2

Question 16 (5 points). Find \( y'' \) where \( y = \frac{x}{x^2 - 4} \).

\[
y' = \frac{(x^2 - 4) \cdot \frac{d}{dx} [x] - (x) \cdot \frac{d}{dx} [x^2 - 4]}{(x^2 - 4)^2} \quad \text{(Quotient Rule)}
\]

\[
y' = \frac{(x^2 - 4 \cdot 1) - (x) \cdot (2x)}{(x^2 - 4)^2} = \frac{x^2 - 4 - 2x^2}{(x^2 - 4)^2} = \frac{-x^2 - 4}{(x^2 - 4)^2}
\]

Section 6.4

Question 17 (5 points). Find \( \frac{dy}{dx} \) using implicit differentiation.

\[
\frac{d}{dx} \left[ (x^3 + y^3)^3 \right] = \frac{d}{dx} \left[ x^3 y^2 \right]
\]

\[ (x^3 + y^3)^3 = x^3 y^2 \]

\[
\Rightarrow 2(x^3 + y^3)(3x^2 + 3y^2 \cdot \frac{dy}{dx}) = x^2 (2y \cdot \frac{dy}{dx}) + y^2 (2x)
\]

\[ 2(x^3 + y^3) (3x^2) + 2(x^3 + y^3) (3y^2) \cdot \frac{dy}{dx} = 2x^2 y \cdot \frac{dy}{dx} + 2xy^2
\]

\[ \Rightarrow \left[ 2(x^3 + y^3) (3x^2) - 2x^2 y \right] \cdot \frac{dy}{dx} = 2x^2 y - 2(x^3 + y^3) (3x^2)
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{2x^2 y - 2(x^3 + y^3) (3x^2)}{2(x^3 + y^3) (3x^2) - 2x^2 y} = \frac{2x^2 y - (x^3 + y^3) (3x^2)}{2 \left[ (x^3 + y^3) (3x^2) - x^2 y \right]}
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{xy^2}{x^2 y^2 + 3x^2 y - 3x^2 y^2}
\]
Section 5.1/5.2/5.3

Question 18. Consider the graph of the function \( f(x) = x^3 + 6x^2 + 2x \).

(a) (3 points) Find all interval(s) on which \( f(x) \) is increasing.

From number line:

\( f(x) \) is increasing on \((-\infty, -4) \) and \((0, \infty) \)

(b) (3 points) Find all interval(s) on which \( f(x) \) is decreasing.

From number line:

\( f(x) \) is decreasing on \((-4, 0) \)

(c) (3 points) Find the coordinates of any point(s) where \( f(x) \) has a local maximum.

Use Second Derivative Test:

Check the critical points: \( x = -4 \) and \( x = 0 \)

\[ f''(x) = \frac{d}{dx} \left[ 3x^2 + 12x \right] = 6x + 12 \]

\[ f''(-4) = (6(-4)) + 12 = -24 + 12 = -12 < 0 \Rightarrow \text{Concave down at } x = -4 \]

At other critical point

\[ f'(0) = (0)^3 + 6(0)^2 + 2 = 0 + 0 + 2 = 2 \]

\[ f''(0) = 6(0) + 12 = 12 > 0 \Rightarrow \text{Concave up at } x = 0 \]

Local Max at \((-4, 34)\)

Local Min at \((0, 2)\)

(d) (3 points) Find the coordinates of any point(s) where \( f(x) \) has a local minimum.

(e) (3 points) Find all interval(s) on which \( f(x) \) is concave up.

\[ f''(x) = 6x + 12 \]

\[ f''(x) = 0 \Rightarrow 6x + 12 = 0 \]

\[ 6x = -12 \Rightarrow x = -2 \]

\[ f''(x) = \frac{d}{dx} \left[ 6x + 12 \right] = 6 > 0 \]

Concave up on \((-\infty, -2)\)

(f) (3 points) Find all interval(s) on which \( f(x) \) is concave down.

From number line,

\( f(x) \) is concave down on \((-\infty, -2)\)

(g) (2 points) Find the coordinates of any inflection points of \( f(x) \).

Concavity changes at \( x = -2 \)

\[ f(-2) = (-2)^3 + 6(-2)^2 + 2 = -8 + 24 + 2 = 18 \]

Inflection point: \((-2, 18)\)
Question 19 (5 points). Find the absolute maximum and absolute minimum values of the function:

\[ f(x) = 4 - 2x - x^2 \]

on the interval \([-2, 2]\). Indicate at which \(x\)-values each of these extrema occur.

**Critical Points**

1. \( f'(x) = 0 \Rightarrow \frac{d}{dx}[4 - 2x - x^2] = 0 \Rightarrow -2 - 2x = 0 \Rightarrow 2x = -2 \Rightarrow x = -1 \)

   \[ f(-1) = 4 - 2(-1) - (-1)^2 = 4 + 2 - 1 = 5 \]

2. \( f'(x) \text{ DNE} \quad \text{No such critical points since } f'(x) = -2 - 2x \text{ is defined for all } x \text{ values} \)

**Endpoints**

\[ f(-2) = 4 - 2(-2) - (-2)^2 = 4 + 4 - 4 = 4 \]

\[ f(2) = 4 - 2(2) - (2)^2 = 4 - 4 - 4 = -4 \]

From ordered values:

- Absolute max. is 5 at \( x = -1 \)
- Absolute min is -4 at \( x = 2 \)

Question 20 (5 points). In the following equation, assume that \( x \) and \( y \) are differentiable functions of \( t \).

Find \( \frac{dx}{dt} \) when \( x = 2, y = 3 \), and \( \frac{dy}{dt} = 2 \).

\[ x^2y^3 = 108 \]

\[ \frac{d}{dt}[x^2y^3] = \frac{d}{dt}[108] \]

\[ \Rightarrow (x^2)(3y^2 \cdot \frac{dy}{dt}) + (y^3)(2x \cdot \frac{dx}{dt}) = 0 \]

\[ \text{Product Rule & Chain Rule} \]

\[ \Rightarrow 3x^2y^2 \cdot \frac{dy}{dt} + 2xy^3 \cdot \frac{dx}{dt} = 0 \]

At given values

\[ \frac{dx}{dt} = \frac{-3x^2y^2 \cdot \frac{dy}{dt}}{2xy^3} \]

\[ \frac{dx}{dt} = \frac{-3(2)^2(3)^2 \cdot 2}{2 \cdot 2 \cdot 3^3} \]

\[ \frac{dx}{dt} = -2 \]

\[ \frac{dx}{dt} = \boxed{-2} \]
Question 21 (5 points). Evaluate the indefinite integral.

\[ \int x^2(x^3-1)^4 \, dx \]

\[ \overset{u-subst}{=} \int u^4 \left( \frac{1}{3} \, du \right) \]

\[ = \frac{1}{3} \int u^4 \, du \]

\[ = \frac{1}{3} \left( \frac{u^5}{5} \right) + C \]

\[ = \frac{1}{15} u^5 + C \]

\[ \overset{reverse \ subst}{=} \frac{1}{15} (x^3-1)^5 + C \]

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Question 22 (5 points). Evaluate the definite integral.

\[ \int_2^3 \frac{1}{x \ln x} \, dx \]

\[ \overset{(u-subst)}{=} \int_2^3 \frac{1}{u} \, du \]

\[ \overset{(changing \ limits)}{=} \int_{\ln 2}^{\ln 3} \frac{1}{u} \, du \]

\[ = \ln |u| \bigg|_{\ln 2}^{\ln 3} \]

\[ = \ln |3| - \ln |2| \]

\[ = \ln \left( \frac{3}{2} \right) \approx 0.461 \]
**Question 23 (5 points).** Barbara knows that she will need to buy a new car in 4 years. The car will cost $15000 by then. How much should she invest now at 8%, compounded quarterly, so that she will have enough to buy a new car? Round to the nearest cent.

\[ A = P \left(1 + \frac{r}{m}\right)^{tm} \Rightarrow P = \frac{A}{\left(1 + \frac{r}{m}\right)^{tm}} \]

Here, \( A = 15000 \), \( r = 0.08 \), \( m = 4 \), \( t = 4 \). So,

\[ P = \frac{15000}{\left(1 + \frac{0.08}{4}\right)^{(4)(4)}} = \frac{15000}{1.02^{16}} = \boxed{10,926.69} \] (to nearest cent)

**Question 24 (5 points).** In the formula \( A(t) = A_0e^{kt} \), \( A(t) \) is the amount of radioactive material remaining from an initial amount \( A_0 \) at a given time \( t \) and \( k \) is a negative constant determined by the nature of the material. A certain radioactive isotope has a half-life of 1300 years. How many years would be required for a given amount of this isotope to decay to 45% of that amount?

**Half-life of 1300 years \( \Rightarrow \) \( A(1300) = \frac{1}{2}A_0 \)**

Solving for \( k \):

\[ \frac{1}{2}A_0 = A_0e^{1300k} \Rightarrow e^{1300k} = \frac{1}{2} \Rightarrow \ln e^{1300k} = \ln \frac{1}{2} \]

\[ 1300k = \ln \frac{1}{2} \Rightarrow k = \frac{\ln \frac{1}{2}}{1300} \] So, \( A(t) = A_0e^{\frac{\ln \frac{1}{2}}{1300}t} \)

Now, find \( t \) when \( A(t) = 0.45A_0 \) (45% of original amount remains)

\[ 0.45A_0 = A_0e^{\frac{\ln \frac{1}{2}}{1300}t} \Rightarrow e^{\frac{\ln \frac{1}{2}}{1300}t} = 0.45 \]

\[ \Rightarrow \frac{\ln \frac{1}{2}}{1300}t = \ln 0.45 \Rightarrow t = \frac{(1300)(\ln 0.45)}{\ln \frac{1}{2}} \approx 1498 \text{ years} \] (to nearest year)
Question 25 (5 points). Find the average rate of change of the function \( y = x^3 + x^2 - 8x - 7 \) between \( x = 0 \) and \( x = 2 \).

\[
\text{Avg. rate of change} = \frac{f(b) - f(a)}{b - a} = \frac{f(2) - f(0)}{2 - 0} = \frac{(2^3 + 2^2 - 8(2) - 7) - (0^3 + 0^2 - 8(0) - 7)}{2}
\]
\[
= \frac{8 + 4 - 16 - 7 - 0 + 0 - 7}{2} = \frac{-11 + 7}{2} = \frac{-4}{2} = -2
\]

Question 26 (5 points). Suppose that the dollar cost of producing \( x \) radios is \( C(x) = 800 + 40x - 0.2x^2 \). Find the marginal cost when 50 radios are produced.

\[
\text{Marginal cost function} = C'(x)
\]
\[
C'(x) = \frac{d}{dx} \left[ 800 + 40x - 0.2x^2 \right] = 40 - 0.4x
\]

When \( x = 50 \) radios:
\[
C'(50) = 40 - 0.4(50) = 40 - 20 = \boxed{20}
\]

Question 27 (5 points). Suppose that \( f(x) = \frac{2}{x} \) and \( g(x) = 2x^3 \). Find \( f(g(x)) \) and \( g(f(x)) \).

i) \( f(g(x)) = f \left( \frac{2}{x^3} \right) = \frac{2}{(2x^3)} = \frac{1}{x^3} \)

ii) \( g(f(x)) = g \left( \frac{2}{x} \right) = 2 \left( \frac{2}{x} \right)^3 = 2 \left( \frac{8}{x^3} \right) = \frac{16}{x^3} \)
**Question 28 (5 points).** The total revenue from the sale of $x$ stereos is given by $R(x) = 2000 \left(1 - \frac{x}{600}\right)^2$.

Find the marginal average revenue.

Average revenue $= \overline{R}(x) = \frac{R(x)}{x} = \frac{2000 \left(1 - \frac{x}{600}\right)^2}{x} = \left(\frac{2000}{x}\right)\left(1 - \frac{x}{600}\right)^2$

Marginal average revenue $= \frac{d}{dx} \left(\frac{2000}{x}\right)\left(1 - \frac{x}{600}\right)^2 = \left(\frac{2000}{x}\right)\left(2\left(1 - \frac{x}{600}\right)(-\frac{x}{600}) - \frac{x}{600}\right) + \left(1 - \frac{x}{600}\right)^2(-2000x^{-2})$

$= \left(1 - \frac{x}{600}\right)[-\frac{4000}{600x} + (1 - \frac{x}{600})(-\frac{2000}{x^2})] = \left(1 - \frac{x}{600}\right)[-\frac{4000}{600x} - \frac{2000}{x^2} + \frac{2000x}{600x^2}]$

$= \left(1 - \frac{x}{600}\right)[-\frac{40}{6x} - \frac{2600}{x^2} + \frac{2000x}{600x^2}] = \left(1 - \frac{x}{600}\right)[-\frac{20}{6x} - \frac{2000}{x^2}] = \left(1 - \frac{x}{600}\right)[-\frac{10}{3x} - \frac{2000}{x^2}]$

$= -\frac{10}{3x} - \frac{2000}{x^2} + \frac{10}{180x} - \frac{2000x}{600x^2} = -\frac{10}{3x} - \frac{2000}{x^2} + \frac{1}{180} + \frac{10x}{3x} = \frac{1}{180} - \frac{2000}{x^2} \quad \text{(Fully Simplified)}$

**Question 29 (5 points).** Find the derivative of the function $y = 5x^3 e^{3x}$.

$\frac{dy}{dx} = (5x^3)(e^{3x}\cdot 3) + (e^{3x})(10x)$ \quad \text{Product Rule}$

$= 15x^2 e^{3x} + 10xe^{3x}$

$= 5xe^{3x}(3x + 2)$

**Question 30 (5 points).** Find the derivative of the function $y = 8^{10x}$.

$\frac{dy}{dx} = 8^{10x} \cdot \ln(8) \frac{d}{dx}[10x] = 8^{10x} \cdot \ln(8) \cdot 10$ \quad \text{Chain Rule}$

$= 10 \ln(8) \cdot 8^{10x}$

**Question 31 (5 points).** Find the derivative of the function $y = \ln(2 + x^2)$.

$\frac{dy}{dx} = \frac{1}{2 + x^2} \frac{d}{dx}[2 + x^2] = \frac{1}{2 + x^2} (2x) = \frac{2x}{2 + x^2}$
Question 32 (5 points). Find the derivative of the function \( y = \log_9 \sqrt{5x+5} \).

\[
\frac{dy}{dx} = \frac{1}{(\ln 9) \sqrt{5x+5}} \cdot \frac{1}{\sqrt{5x+5}}
\]

Chain Rule

\[
= \frac{5}{(\ln 9)(5x+5)^{\frac{3}{2}}}
\]

= \frac{5}{2(\ln 9)(5x+5)}

Question 33 (5 points). Maximize \( Q = xy^2 \), where \( x \) and \( y \) are positive numbers and \( x + y^2 = 8 \).

\( x + y^2 = 8 \Rightarrow y^2 = 8 - x \)

So, \( Q = xy^2 = x(8-x) = 8x - x^2 \)

Maximize \( Q(x) = 8x - x^2 \) on domain \( x > 0 \) (must be positive)

Critical Nos:

i) \( Q'(x) = 0 \Rightarrow 8 - 2x = 0 \)
\( 2x = 8 \)
\( x = 4 \)

When \( x = 4 \), \( y^2 = 8 - x = 8 - 4 = 4 \)
\( y^2 = 4 \Rightarrow |y| = 2 \)
\( y = \pm 2 \)

But, \( y > 0 \Rightarrow y = 2 \) (\( y \) must be positive too)

ii) \( Q'(x) \) DNE \( \Rightarrow \) no such pts.

(since \( 8 - 2x \) is a polynomial)

So, \( Q \) is maximized when \( x = 4 \) and \( y = 2 \)

\( \text{Max value of } Q \) is \( Q = xy^2 = (4)(2)^2 = (4)(4) = 16 \)

(Note: \( Q''(x) = \frac{d}{dx}[8 - 2x] = -2 \), Critical point at \( x = 4 \) is a maximum (by 2nd Derivative Test))
Question 34 (5 points). Evaluate the integral.

\[ \int (8x^2 + x^{-4}) \, dx \]

\[ = 8 \int x^2 \, dx + \int x^{-4} \, dx \]

\[ = 8 \cdot \frac{x^3}{3} + \frac{x^{-3}}{-3} + C \]

\[ = \frac{8}{3} x^3 - \frac{1}{3} x^{-3} + C \]

Question 35 (5 points). Evaluate the integral.

\[ \int x^2(3x + x^{-3}) \, dx \]

\[ = \int (3x^3 + x^{-1}) \, dx \]

\[ = 3 \int x^3 \, dx + \int x^{-1} \, dx \]

\[ = 3 \left( \frac{x^4}{4} \right) + \ln |x| + C \]

\[ = \frac{3}{4} x^4 + \ln |x| + C \]

Question 36 (5 points). Approximate the area under the graph of \( f(x) = 3x^2 - 2 \) and above the x-axis from \( x = 1 \) to \( x = 5 \) by using 4 equal-width rectangles and right endpoints.

\[ \Delta x = \frac{b-a}{n} = \frac{5-1}{4} = \frac{4}{4} = 1 \]

By rectangle approximation:

\[ A \approx \sum_{i=1}^{4} f(x_i) \Delta x \]

\[ = f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) \]

\[ = [3(2)^2 - 2] + [3(3)^2 - 2] + [3(4)^2 - 2] + [3(5)^2 - 2] \]

\[ = 10 + 25 + 46 + 73 \]

\[ = 154 \]